CORRESPONDENCE

Comments on "On the Structure and Formation of UTLS PV Dipole/Jetlets in Tropical Cyclones by Convective Momentum Surges"

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ABSTRACT: This comment on Hitchman and Rowe first deepens their introduction by distinguishing adiabatic and diabatic tilting of vorticity. Then, it strengthens their interpretation by emphasizing that momentum must be vertically transported with reference to isentropic levels to yield the potential vorticity (PV) dipoles. Moreover, it points out a flaw in their PV budget analysis and proposes a remedy for the flaw. Their convective momentum transport paradigm and the vorticity tilting paradigm reinterpret the same physical process. However, they counted one physical process twice by associating the two paradigms with two different terms. As an attempt to remedy the flaw, this comment associates the reinterpretation of the two paradigms with a transformation of the PV equation; their paradigm corresponds to a flux form. With the proposed remedy, their paradigm can be more easily translated to advances in convective parameterization because of its horizontal locality.

KEYWORDS: Convection; Dynamics; Fluxes; Heating; Momentum; Potential vorticity

Hitchman and Rowe (2019, hereafter HR19) proposed a reinterpretation for the formation of horizontal dipoles of potential vorticity (PV) due to interactions between convection and horizontal vorticity. Well known in mesoscale meteorology, horizontal dipoles of vertical vorticity are generally due to tilting of vorticity from horizontal to vertical by differential vertical motion (e.g., Chagnon and Gray 2009; Davies-Jones 1984). Whether the vorticity tilting yields horizontal dipoles of PV depends on the nature of the vertical motion, illustrated in Fig. 1. For simplicity, consider an original configuration in which isentropic surfaces and a vorticity tube are in parallel (Fig. 1a), associated with no PV. Davies-Jones (1984) considered adiabatic inviscid flow and concluded that the tilting of vorticity does not change the PV field; the isentropic surfaces and the vorticity tube are still in parallel after adiabatic tilting (Fig. 1b). Chagnon and Gray (2009) demonstrated that tilting of vorticity from along isentropic to cross isentropic by differential diabatic heating or cooling yields a horizontal dipole of PV (Fig. 1c). Hitchman and Rowe (2017) confirmed the theory of Chagnon and Gray (2009) and discussed the PV equation, which can be written using the notation of Vallis (2017):

$$\frac{DQ}{Dt} = Q\frac{\partial\dot{\theta}}{\partial\theta} + \frac{\boldsymbol{\omega}_{||}}{\rho}\cdot\nabla_{\theta}\dot{\theta} + \frac{\nabla\theta}{\rho}\cdot(\nabla\times\mathbf{F}), \tag{1a}$$

$$Q \equiv \frac{\boldsymbol{\omega}_a}{\rho}\cdot\nabla\theta, \tag{1b}$$

where the variables are defined as follows: Q is PV, $\boldsymbol{\omega}_a$ is absolute vorticity, $\boldsymbol{\omega}_{||}$ is the component of absolute vorticity parallel to isentropic surfaces, ρ is density, θ is potential temperature, $\dot{\theta}$ is diabatic heating or cooling, and $\mathbf{F}$ is the viscous force and apparent force due to subgrid-scale parameterization. Equation (1a) states that material temporal tendency of PV equals to sum of three types of PV sources and sinks. The first and second terms on the rhs, pointed out by Hitchman and Rowe (2017), correspond to diabatic (or latent) stretching and tilting in Chagnon and Gray (2009). The third term is associated with viscosity and subgrid-scale processes. Introducing PV dipoles, HR19 confused Davies-Jones (1984) with Chagnon and Gray (2009); only the latter is appropriate. Despite this minor confusion, HR19 proposed the convective momentum transport paradigm, which intrigues me and is the focus of this comment.

The convective momentum transport paradigm describes the same physical process as the vorticity tilting paradigm (Fig. 1) but reinterprets it (HR19). In the original configuration, there is vertical shear of horizonal velocity; on the velocity ring in the middle of Fig. 1a, the wind goes into the paper at the lower isentropic level and comes out of the paper at the upper isentropic level. Assuming momentum conservation, a localized convective updraft transports the into-the-paper momentum originally at the lower level upward, whereby it injects the momentum onto a level that originally has less into-the-paper momentum, yielding an into-the-paper "jetlet" (Figs. 1b,c). HR19 stated that the jetlet corresponds to a positive PV to the left and a negative PV to the right (i.e., a PV dipole). This comment points out an ambiguity in this statement; what does the level refer to? If the level refers to the height level, the vertical transport of momentum does not necessarily yield a PV dipole; Davies-Jones (1984) gives a counterexample (Fig. 1b). To strengthen HR19's interpretation, this comment emphasizes that the level refers to the isentropic level. HR19 mentioned diabatic heating as the fuel for the convective updraft, but Fig. 1c illustrates that diabatic heating plays a more essential role than the fuel. Diabatic heating transports momentum upward *with reference to isentropic levels*. Diabatic heating can yield the PV dipole even

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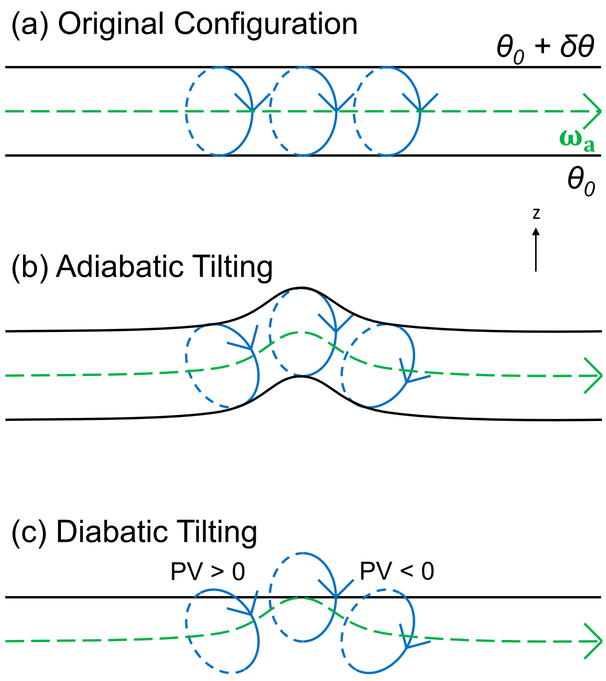

FIG. 1. A schematic diagram illustrating changes in isentropic surfaces (black) and a vorticity tube (vorticity in green and velocity rings perpendicular to it in blue) that are (a) originally in parallel and have undergone (b) adiabatic tilting and (c) diabatic tilting.

without any vertical velocity; in such a case, the vorticity tube in Fig. 1a remains unchanged, but the isentropic surfaces hollow due to diabatic heating. In spite of this ambiguity, HR19 pointed out an important advantage of the convective momentum transport paradigm; that is, it is horizontally local.

The horizontal locality of the convective momentum transport paradigm makes it convenient for qualitative diagnosis (HR19). Given the vertical profiles of horizontal wind and diabatic heating or cooling (not vertical velocity as explained in the previous paragraph) at a single column, assuming momentum conservation, one can calculate the jetlet; that is, the wind vector difference from the target isentropic level to the parcel source level. The jetlet corresponds to a PV increase to the left and a decrease to the right (HR19), as if, to my intuition, PV fluxes from the right to the left; such a PV flux is not advective and is rigorously defined in the next paragraph. HR19 suggested that the jetlet is generally maximized in convective updrafts, so the jetlet diagnosis at convective updrafts indicates the orientation of PV dipoles; unlike the vorticity tilting paradigm needs a 3D view. However, in terms of PV budget analysis, HR19 was flawed. HR19 argued that the convective momentum transport corresponds to the third term on the rhs of Eq. (1a). In the context of HR19, the convective momentum transport is a resolved advective transport but not an unresolved subgrid-scale transport, but this term does not account for any resolved advection. Proof by contradiction also shows the flaw in that argument. The paradigms of convective momentum transport and vorticity tilting describe the same physical process (HR19), and the vorticity tilting corresponds to the second term on the rhs of Eq. (1a) (Hitchman and Rowe 2017). Consequently, if that argument were true, the physical process would be counted twice by both the second and third terms. A PV budget analysis counting one process twice is flawed. The following paragraph proposes a remedy for the flaw.

The abovementioned PV-flux analogy hints me to explore nonunique flux forms of the PV equation. I found a flux form proposed by Haynes and McIntyre (1990) insightful, which can be written with Vallis' (2017) notation:

$$\frac{\partial \rho Q}{\partial t} = -\nabla \cdot \mathbf{J}, \tag{2a}$$

$$\mathbf{J} = \rho Q \mathbf{v}_{\perp} + \rho Q \mathbf{v}_{||} - \dot{\theta}\boldsymbol{\omega}_{||} - \mathbf{F} \times \nabla\theta, \tag{2b}$$

where $\mathbf{v}_{\perp}$ denotes rate of normal displacement of an isentropic surface, and $\mathbf{v}_{||}$ denotes the component of wind velocity parallel to isentropic surfaces. Equation (2a) states that local tendency of PV density equals to convergence of sum of four types of PV density flux in Eq. (2b), namely, displacement flux, advective flux, diabatic flux, and viscos flux. The displacement flux is perpendicular to the isentropic surface but does not permeate it. The latter three fluxes are along the isentropic surface. The advective flux corresponds to both advection and diabatic stretching of PV; $-\nabla \cdot (\rho Q \mathbf{v}_{||}) = -\mathbf{v}_{||} \cdot \nabla(\rho Q) - \rho Q \nabla \cdot \mathbf{v}_{||}$, and the latter term means along-isentropic concentration and implies cross-isentropic stretching assuming continuity. This comment argues that the convective momentum transport paradigm (HR19) corresponds to the diabatic flux of PV density ($-\dot{\theta}\boldsymbol{\omega}_{||}$). Given a cross-isentropic wind shear, associated with along-isentropic vorticity, $\boldsymbol{\omega}_{||}$ (Fig. 1a), the diabatic flux term suggests that diabatic heating ($\dot{\theta} > 0$) injects a jetlet toward the left of the vorticity into the isentropic layer, corresponding to a PV density flux opposing (the negative sign) the vorticity (Fig. 1c). Consistent with HR19, the diabatic flux term is horizontally local and can be used to diagnose the orientation of PV dipoles. As a proposed remedy for the flaw in HR19, instead of counting different terms to account for the same physical process, the diabatic flux term in the local flux form [Eq. (2)] is a transformation of the diabatic tilting term in the material form of sources and sinks [Eq. (1)]; like the convective momentum transport paradigm is a reinterpretation of the vorticity tilting paradigm. Harvey et al. (2020) suggested that the diabatic flux term explains the negative PV often observed on the equatorward flank of North Atlantic jet streams.

With the diabatic flux term in Eq. (2), the convective momentum transport paradigm (HR19) can benefit not only qualitative but also quantitative diagnoses. In comparison to the vorticity tilting paradigm, because of the horizontal locality, the convective momentum transport paradigm can be more easily translated into advances in convective parameterization, for which treatment of subgrid-scale convective momentum transport is subject to ongoing research (e.g., Rio et al. 2019; Woelfle et al. 2018).

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